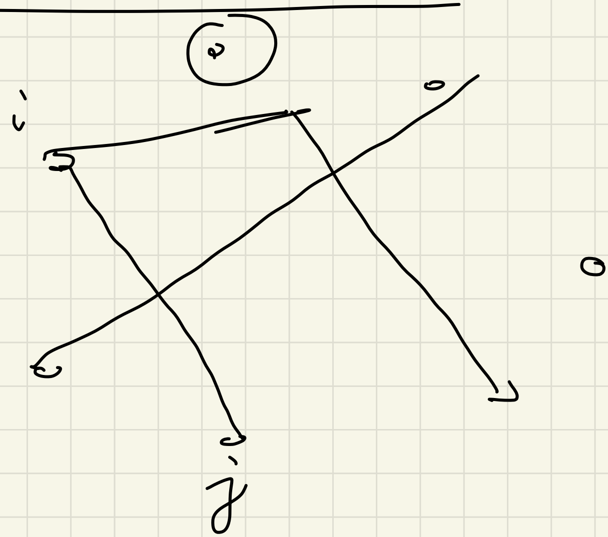
March 3rd: 2022:

Semidefinite programs & Combinatorial opt:

Lovasz:

- Shannon Capacity:

V



- Sending

- length -1- $\alpha(G)$;

- length ②:

$G \times G$: (v_i, v_j), $v_i \in V$
$v_j \in V$;

V W

G, H:

(v, w) (v', w').

$= (v, v') \in E$, & $(w, w') \in E$

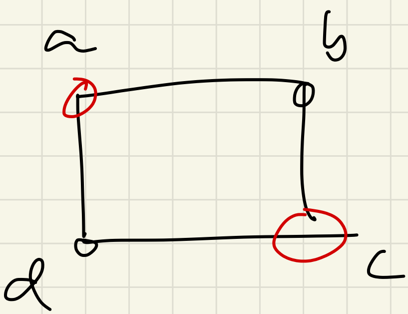
or: $v = v'$ & $w - w' \in E(H)$,
$w = w'$ & $v - v' \in E(G)$;

$\left.\begin{array}{l} v_1 v_2 \dots v_k \\ w_1 w_2 \dots w_k \end{array}\right\}$ $\exists i$, $v_i \neq w_i$. & $v_i \neq w_i$

$$\left[\alpha(G^k)\right]^{1/k}$$

$\uparrow$

Shannon capacity of G;



a, c

b, d

a, b

Shannon capacity ≥ 2.

a c a

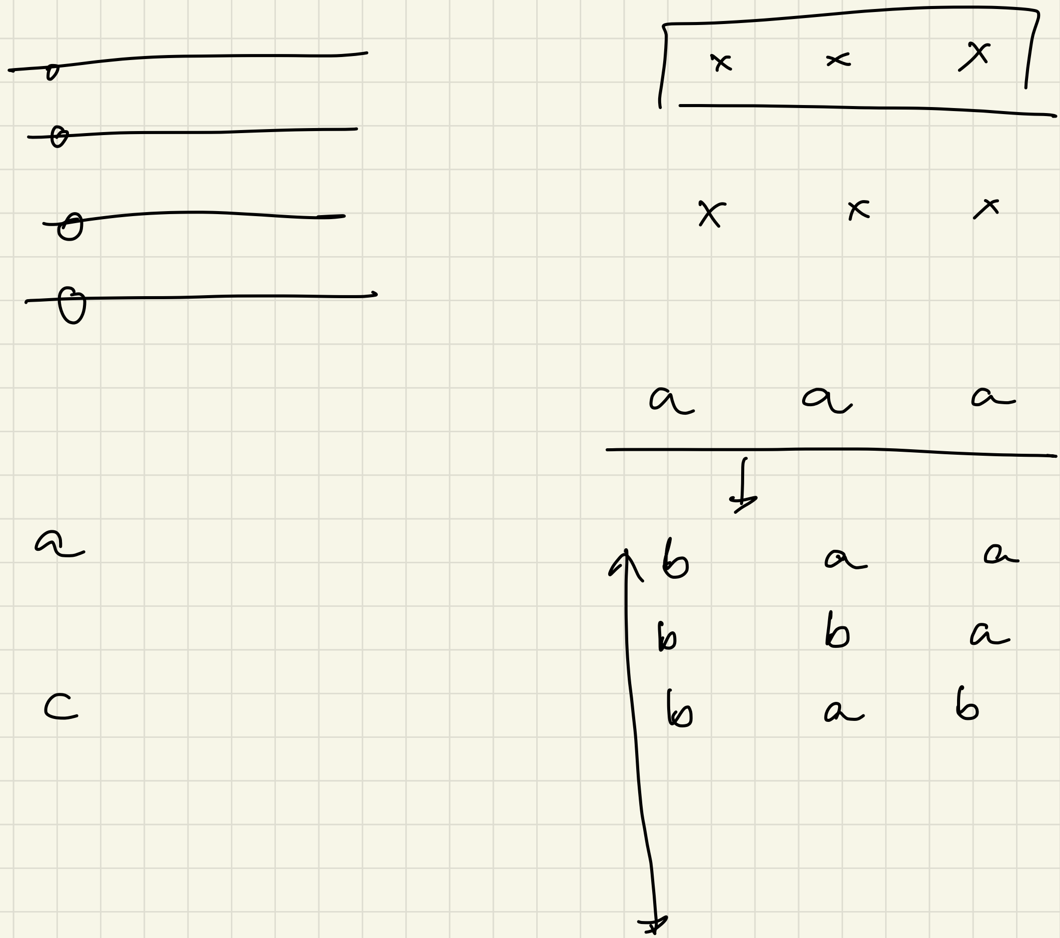
4^k

b c a   b d a

a c a.   d c a.

a b a

a d a



- Take a valid set of words.

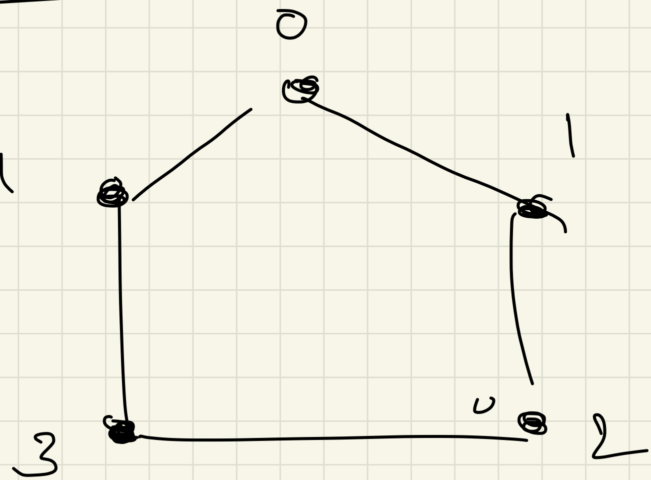
For each word, 2^k words are allowed $\begin{pmatrix} a \rightleftarrows b \\ c \mapsto d \end{pmatrix}$;

No two words in S, disallow same word;

$\# S \le \frac{4^k}{2^k} = 2^k$;

$\therefore$ Shannon cap $= 2$;

C



C^2:

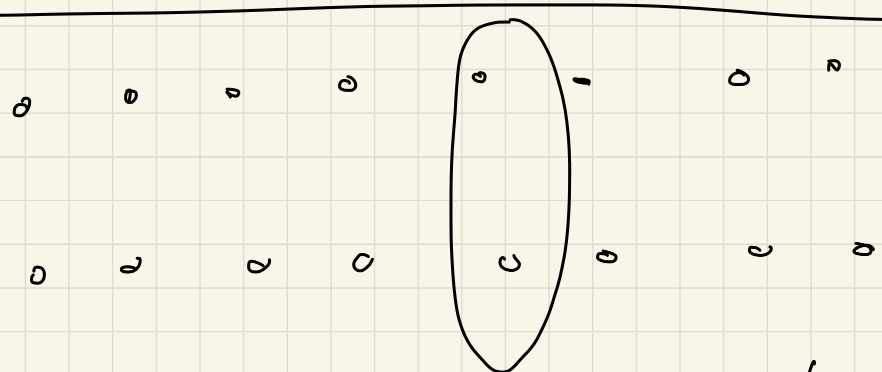
	0	1	2	3	4
valid $\longrightarrow$	00	12	24	31	43.

$\alpha(C^2) = 5$;

G; $\quad G^k$;

$\alpha(G)$.

$\alpha(G^k) \geq (\alpha(G))^k$; $\quad \{ \quad 3,$



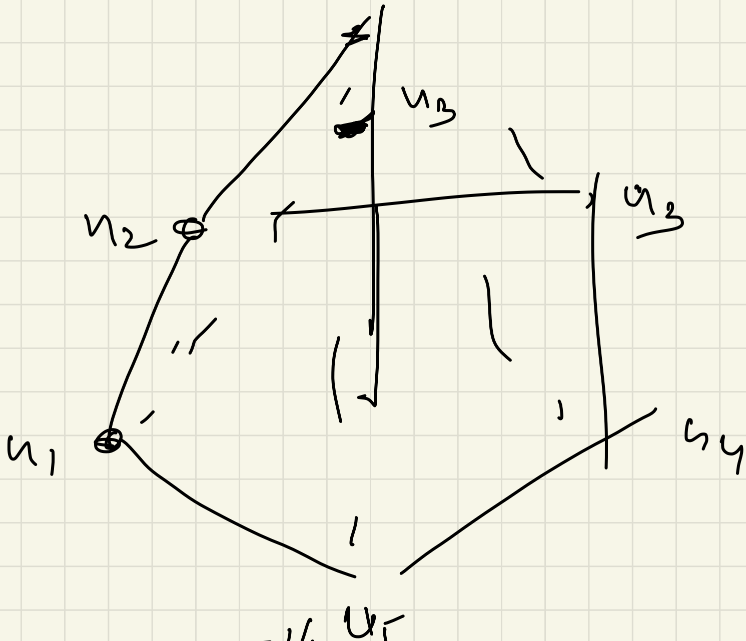
$\therefore \alpha(C^{2k}) \geq (\alpha(C^2))^k$

$\geq (5)^k$

$= (\sqrt{5})^{2k}$

$(\alpha(C^{2k}))^{1/2k} \geq (\sqrt{5})$;

- This is a upper bound;



$\cos\theta = 5^{-1/4}$

$= C^5 \rightsquigarrow$ a unit vector in $\mathbb{R}^3$;

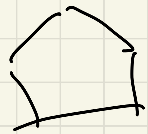
$u_1 \otimes u_1$, $v_1 \otimes u_2$, - - -

$u_1 \cdot (a, b, c)$ $u_2 = (d, e, f)$

$$u_1 \otimes u_2 = (ad, ae, af, bd, be, bf, cd, ce, cf).$$

$\mathbb{R}^3$

$$[u_1 \otimes u_2] \cdot [v_1 \otimes v_2]$$



$$= (u_1^T \cdot v_1) \cdot (u_2^T \cdot v_2)$$

= Take any stable set S in $\mathbb{C}^n$

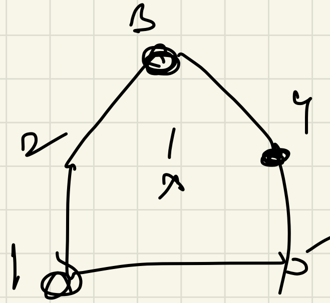
$\{v_i : i \in S\}$ — unit vectors

to $v_i \in S$;

$$\sum_{i \in S} (e^T v_i)^2 \leq 1 ,$$

$$e^T v_i = |e||v_i| \cos\theta$$

$$= \left(5^{-1/4}\right)$$



$$\sum e^T v_i = |S| \cdot 5^{-}$$

$$(u_{i_1} \otimes u_{i_2} \otimes \cdots \otimes u_{i_k})$$

$$(v_{i_1} \otimes v_{i_2} \otimes \cdots \otimes v_{i_k})$$

In (C^k):

C^1: $u_i^T e_i = 5^{-1/4}$.

C^2: $\left(5^{-2/4}\right)$;

In $\underline{\mathbb{C}^2}$: $(i,j) \rightsquigarrow |u_i \otimes u_j\rangle$

$$(u_i \otimes u_j) \cdot (e_1 \otimes e_1)$$

$$= (u_i \otimes e_1) \cdot (u_j \otimes e_1)$$

$$= (5^{-1/4}) \cdot (5^{-1/4})$$

$$= 5^{-2/4}.$$

In $\underline{\mathbb{C}^{2k}}$:

In $\underline{\mathbb{C}^{2k}}$: Can assign unit vectors to vertices so that $e_1^T u_i = \underline{\underline{5^{-k/4}}}$.

$$|S|\left(5^{-k/4}\right)^2 \le \sum_{i \in S} (e_1^T u_i)^2 \le 1$$

$$\therefore \quad |S| \le \left(5^{k/2}\right).$$

$\therefore$ we have assigned to all $v \in \mathbb{R}^k$, unit vectors, non-adj are $\perp$;

S; an indep set;

$$|S| \cdot 5^{-k/2} \le \sum_{u_i \in S} |e_1^T u_i|^2 \le 1.$$

$$u_i = u_{i_1} \otimes \cdots \otimes u_{i_k}$$

$$\boxed{e_1 = e_1 \otimes e_1 \quad \otimes e_1}$$

$u_i^T e_1$: $\left(5^{-1/4}\right)\left(5^{-1/4}\right)\cdots\left(5^{-1/4}\right)$

$$= 5^{-k/4}.$$

$\therefore$ Here: $\left(5^{-k/2}\right)|s| \leq 1$;

$$\therefore |s| \leq 5^{k/2}.$$